7^{TH} Yasinskyi Geometry Olympiad
Grade 8

1. Let O be the circumcenter of triangle ABC and the line AO intersects segment BC at point T. Assume that lines m and ℓ passing through point T are perpendicular to AB and AC respectively. If E is the point of intersection of m and OB and F is the point of intersection of ℓ and OC, prove that $BE = CF$.

(*Oleksii Karliuchenko*)

2. Let I be the incenter of triangle ABC. K_1 and K_2 are the points on BC and AC respectively, at which the inscribed circle is tangent. Using a ruler and a compass, find the center of the inscribed circle for triangle CK_1K_2 in the minimal possible number of steps (each step is to draw a circle or a line).

(*Hryhorii Filippovskyi*)

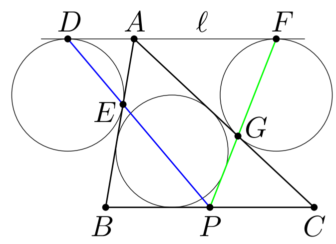
3. ABC is a right triangle with $\angle C = 90^\circ$. Let N be the middle of arc BAC of the circumcircle and K be the intersection point of CN and AB. Assume T is a point on a line AK such that $TK = KA$. Prove that the circle with center T and radius TK is tangent to BC.

(*Mykhailo Sydorenko*)

4. ABC is an acute triangle and AD, BE and CF are the altitudes, with H being the point of intersection of these altitudes. Points A_1, B_1, C_1 are chosen on rays AD, BE and CF respectively such that $AA_1 = HD$, $BB_1 = HE$ and $CC_1 = HF$. Let A_2, B_2 and C_2 be midpoints of segments A_1D, B_1E and C_1F respectively. Prove that H, A_2, B_2 and C_2 are concyclic.

(*Mykhailo Barkulov*)

5. Let ABC be a triangle and ℓ be a line parallel to BC that passes through vertex A. Draw two circles congruent to the circle inscribed in triangle ABC and tangent to line ℓ, AB and BC (see picture). Lines DE and FG intersect at point P. Prove that P lie on BC if and only if P is the midpoint of BC.



(*Mykhailo Plotnikov*)

6. Let ABC be an isosceles triangle with $\angle BAC = 108^\circ$. The angle bisector of the $\angle ABC$ intersects the circumcircle of a triangle ABC at the point D. Let E be a point on segment CB such that $AB = BE$. Prove that the perpendicular bisector of CD is tangent to circumcircle of a triangle ABE.

(*Bohdan Zheliabovskyi*)

December 10, 2023

7^{TH} Yasinskyi Geometry Olympiad
Grade 9

1. Let BD and CE be the altitudes of triangle ABC that intersect at point H. Let F be a point on side AC such that $FH \perp CE$. The segment FE intersects the circumcircle of triangle CDE at the point K. Prove that $HK \perp EF$.

(*Matthew Kurskyi*)

2. Let BC and BD be the tangent lines to the circle with diameter AC. Let E be the second point of intersection of line CD and the circumscribed circle of triangle ABC. Prove that $CD = 2DE$.

(*Matthew Kurskyi*)

3. Let I be the center of the inscribed circle of the triangle ABC. The inscribed circle is tangent to sides BC and AC at points K_1 and K_2 respectively. Using a ruler and a compass, find the center of excircle for triangle CK_1K_2 which is tangent to side CK_2, in at most 4 steps (each step is to draw a circle or a line).

(*Hryhorii Filippovskyi, Volodymyr Brayman*)

4. Let BE and CF be the altitudes of acute triangle ABC. Let H be the orthocenter of ABC and M be the midpoint of side BC. The points of intersection of the mid-perpendicular line to BC with segments BE and CF are denoted by K and L respectively. The point Q is the orthocenter of triangle KLH. Prove that Q belongs to the median AM.

(*Bohdan Zheliabovskyi*)

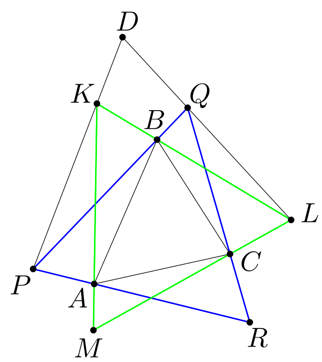
5. Let I be the center of the cirlce inscribed in triangle ABC. The inscribed circle is tangent to side BC at point K. Let X and Y be points on segments BI and CI respectively, such that $KX \perp AB$ and $KY \perp AC$. The circumscribed circle around triangle XYK intersects line BC at point D. Prove that $AD \perp BC$.

(*Matthew Kurskyi*)

6. An acute triangle ABC is surrounded by equilateral triangles KLM and PQR such that its vertices lie on the sides of these equilateral triangle as shown on the picture. Lines PK and QL intersect at point D. Prove that

$$\angle ABC + \angle PDQ = 120^\circ.$$

(*Yurii Biletskyi*)



December 10, 2023

7^{TH} Yasinskyi Geometry Olympiad

Grade 10-11

1. Two circles ω_1 and ω_2 are tangent to line ℓ at the points A and B respectively. In addition, ω_1 and ω_2 are externally tangent to each other at point D. Choose a point E on the smaller arc BD of circle ω_2. Line DE intersects circle ω_1 again at point C. Prove that $BE \perp AC$. (*Yurii Biletskyi*)

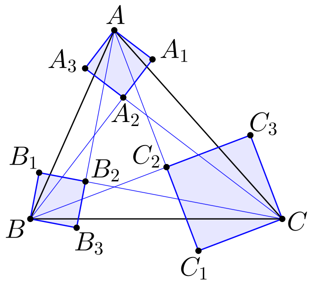
2. Let I be the center of the circle inscribed in triangle ABC which has $\angle A = 60^\circ$ and the inscribed circle is tangent to the side BC at point D. Choose points X and Y on segments BI and CI respectively, such than $DX \perp AB$ and $DY \perp AC$. Choose a point Z such that the triangle XYZ is equilateral and Z and I belong to the same half plane relative to the line XY. Prove that $AZ \perp BC$. (*Matthew Kurskyi*)

3. Let ABC be an acute triangle. Squares $AA_1A_2A_3$, $BB_1B_2B_3$ and $CC_1C_2C_3$ are located such that the lines A_1A_2, B_1B_2, C_1C_2 pass through the points B, C and A respectively and the lines A_2A_3, B_2B_3, C_2C_3 pass through the points C, A and B respectively. Prove that

(a) the lines AA_2, B_1B_2 and C_1C_3 intersect at one point.

(b) the lines AA_2, BB_2 and CC_2 intersect at one point.

(*Mykhailo Plotnikov*)



4. Pick a point C on a semicircle with diameter AB. Let P and Q be two points on segment AB such that $AP = AC$ and $BQ = BC$. The point O is the center of the circumscribed circle of triangle CPQ and point H is the orthocenter of triangle CPQ. Prove that for all posible locations of point C, the line OH is passing through a fixed point. (*Mykhailo Sydorenko*)

5. Let ABC be a scalene triangle. Given the center I of the inscribe circle and the points K_1, K_2 and K_3 where the inscribed circle is tangent to the sides BC, AC and AB. Using only a ruler, construct the center of the circumscribed circle of triangle ABC. (*Hryhorii Filippovskyi*)

6. Let ABC be a scalene triangle. Let ℓ be a line passing through point B that lies outside of the triangle ABC and creates different angles with sides AB and BC. The point M is the midpoint of side AC and the ponts H_a and H_c are the bases of the perpendicular lines on the line ℓ drawn from points A and C respectively. The circle circumscribing triangle MBH_a intersects AB at the point A_1 and the circumscribed circle of triangle MBH_c intersects BC at point C_1. The point A_2 is symmetric to the point A relative to the point A_1 and the point C_2 is symmetric to the point C_1 relative to the point C_1. Prove that the lines ℓ, AC_2 and CA_2 intersect at one point. (*Yana Kolodach*)

December 10, 2023